**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

Ans : Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - 45)/8.0

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 2 6.6% or **0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans : **False** If more employees are older than 44, this will shift the*μ*  towards 44 with considering sd. Which is not possible as *μ* is given 38 with sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: **True**

Probability of employees less than age of 30 = Pr(X<30).

Z = (X -*μ*)/ = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48 i.e 36 employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). 2X1 will just scale the normal distribution by 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Ans: x=σ[z] + µ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = **151.4**

Z(+0.005)\*20+100 = (-2.57)\*20+100 = **48.6**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: **Profit1+profit2~N(5+7,3 2+42)= Profit~N(12,5)**

1. Range = 12-19.6×5,12+ 1.96 ×5

=$2.2, $22.8

=Rs.99, Rs.1026

B) P(Z<=(p-12)/5)=0.05

p-12/5= -1.644

p=12 – 8.22 = $3. 78 = rs.170.1

C) When profit is less than 0 then loss

p-12/5= -1.644

p= 12-8.22 - $3.78 = Rs.170.1